A CAT Tree

CSE 250
Lecture 27

Red-Black Trees

BST Operation Costs

Operation	Runtime
find	O(d)
insert	O(d)
remove	O(d)

Red-Black Trees

- Color each node red or black
  1) # of black nodes from each empty to root must be identical
  2) Parent of a red node must be black

- On Insertion (or deletion)
  - Inserted node is red (won't change # of black nodes)
  - "Repair" violations of rule 2 by rotating or recoloring
    - Repairs guarantee rule 1 is preserved

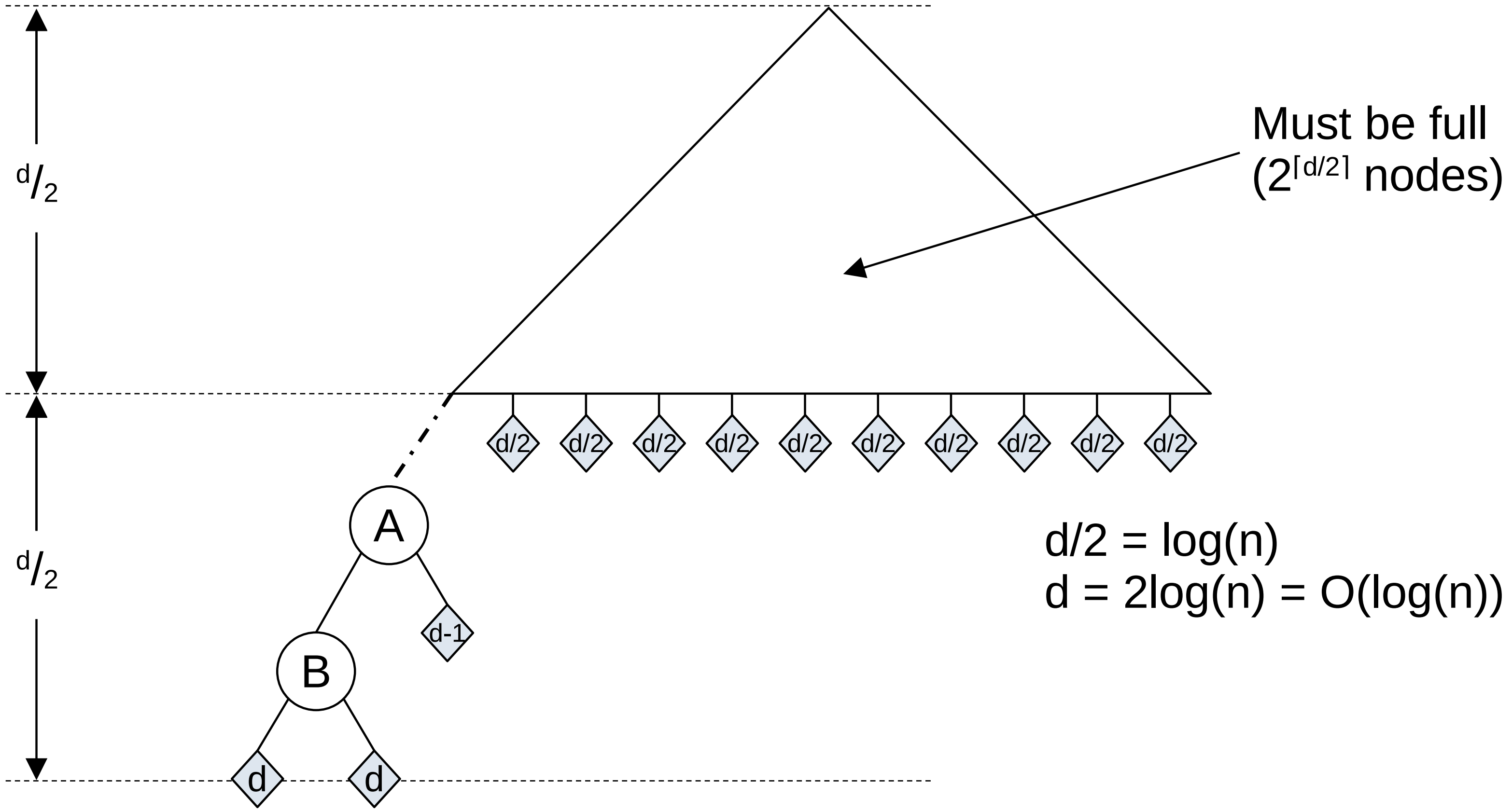
Red-Black Trees

- # of black nodes on a path from root to leaf is the same
  - Call this number (for a given tree) **B**
- Each red node must have a black parent
  - What's the longest possible path from the root to a leaf?

    2B (Black, Red, Black, Red, Black, Red, ...)
  - What's the shortest possible path from the root to a leaf?

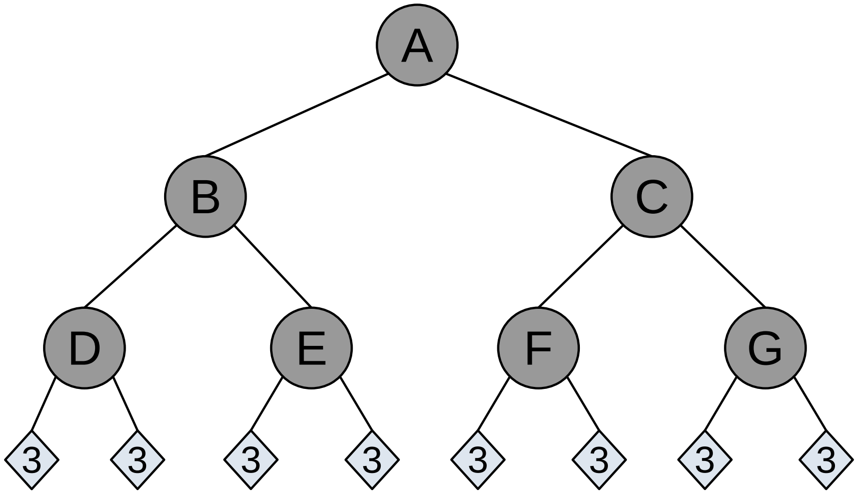
    B (Black, Black, Black, ...)

Balancing Empty Node Depth

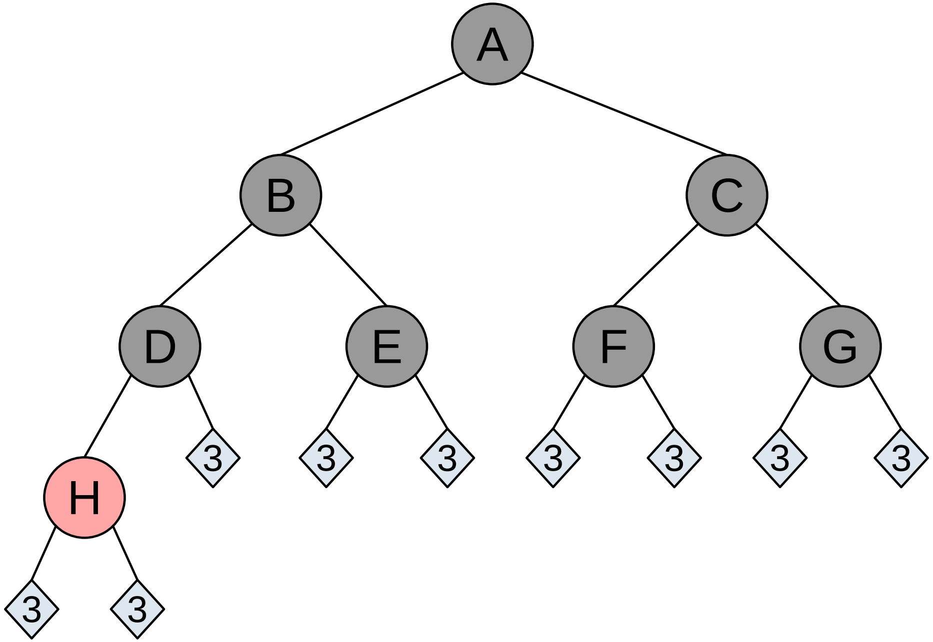


$d/2 = \log(n)$
$d = 2\log(n) = O(\log(n))$

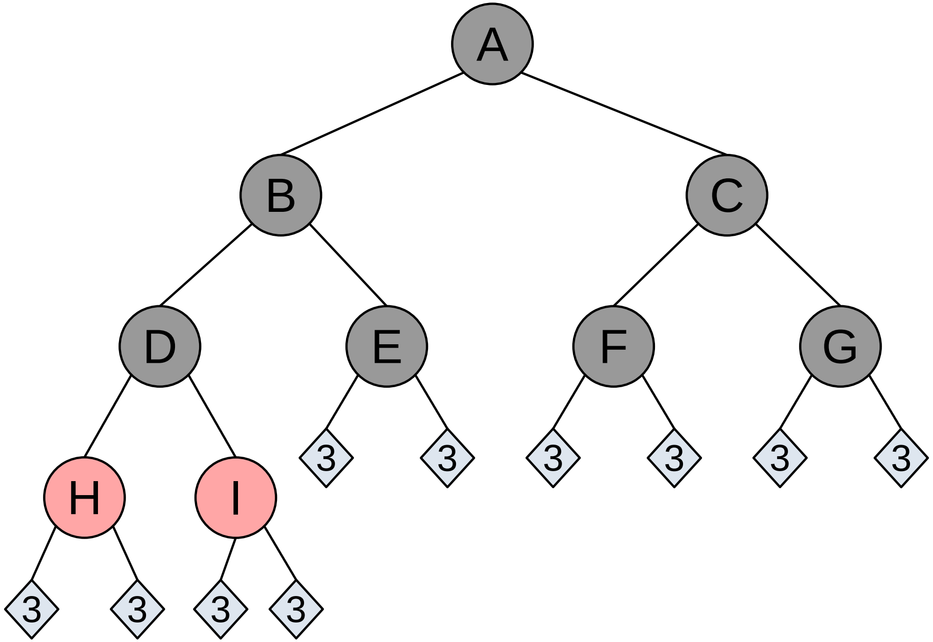
Red-Black Trees



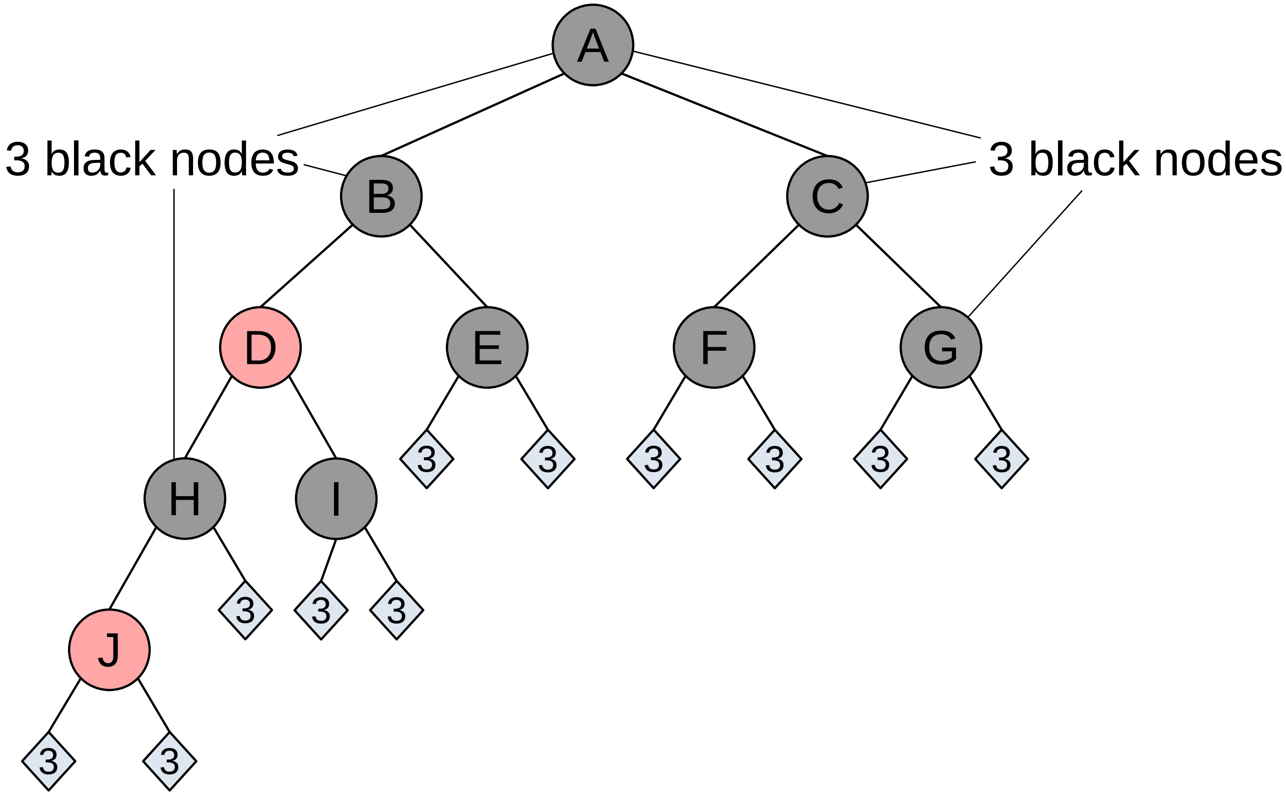
Red-Black Trees



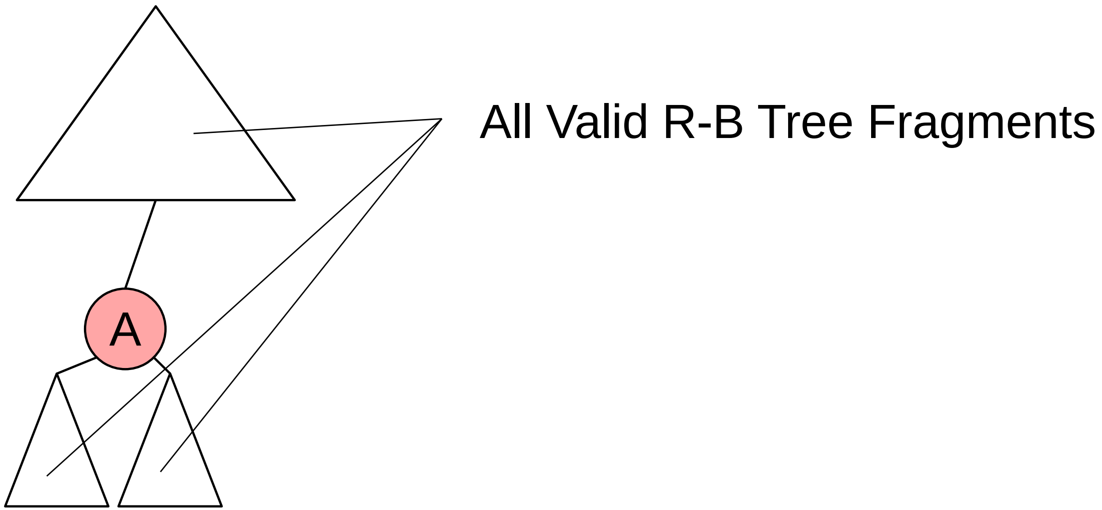
Red-Black Trees



Red-Black Trees



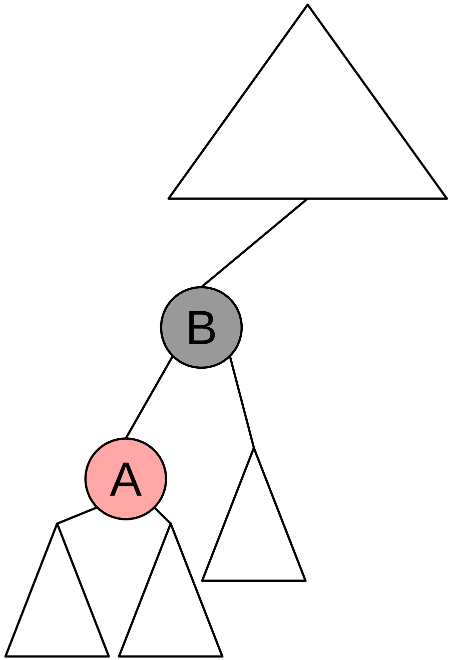
Red-Black Trees



Repair A

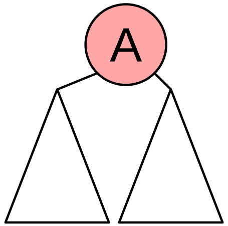
Red-Black Trees

Case 1: All Good!



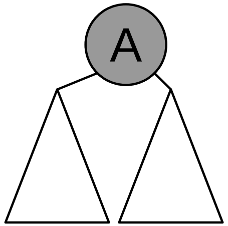
Red-Black Trees

Case 1b: All Good!



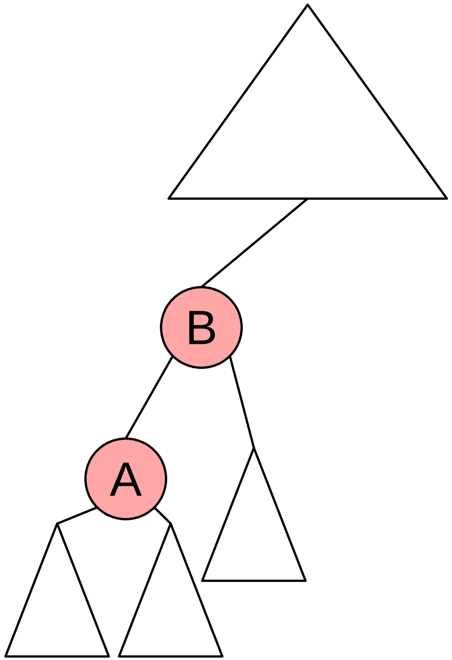
Red-Black Trees

Case 1b: All Good!



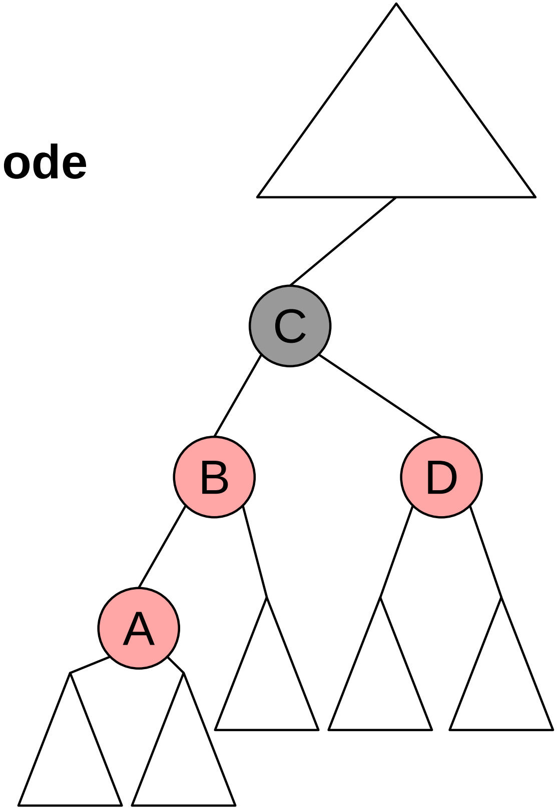
Red-Black Trees

Problem!



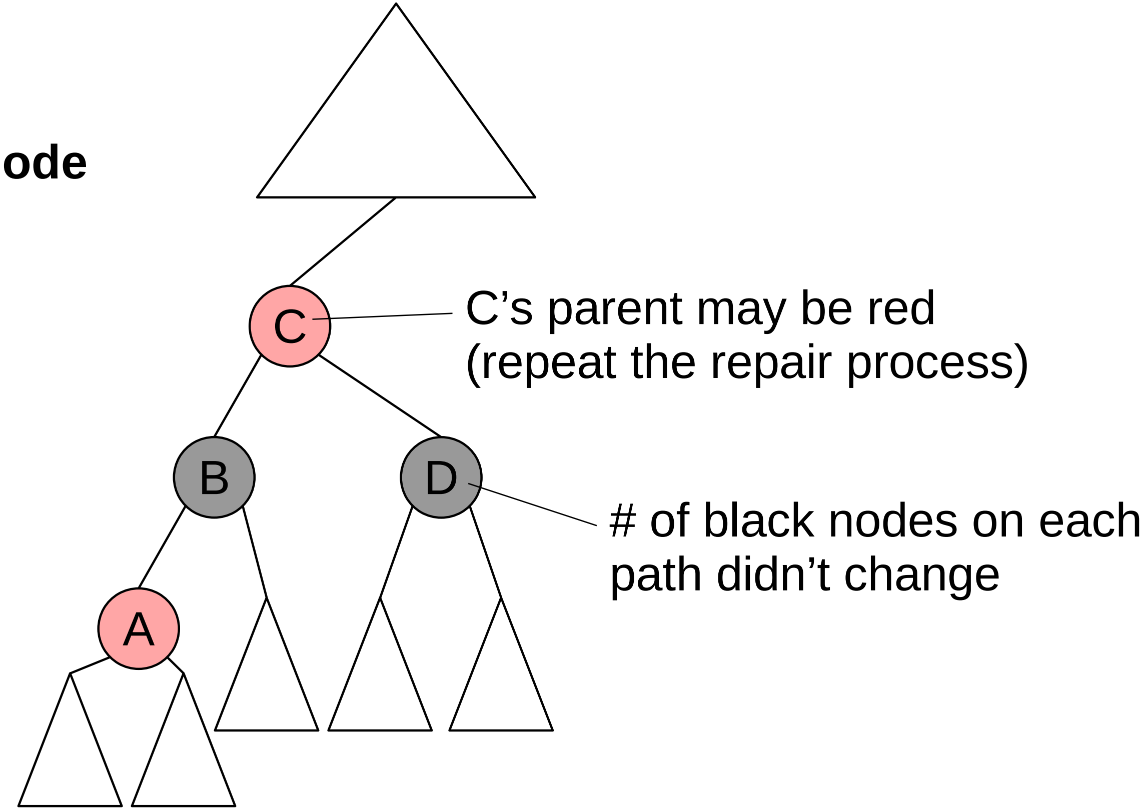
Red-Black Trees

Case 2: Split Black Node



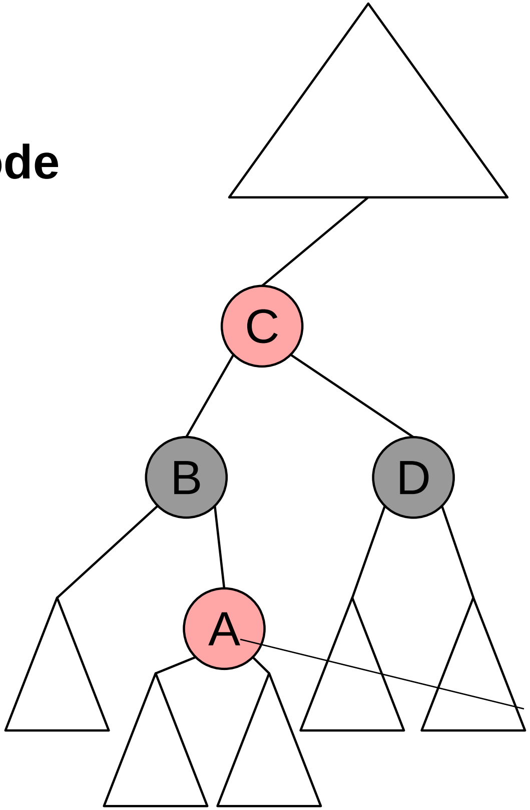
Red-Black Trees

Case 2: Split Black Node



Red-Black Trees

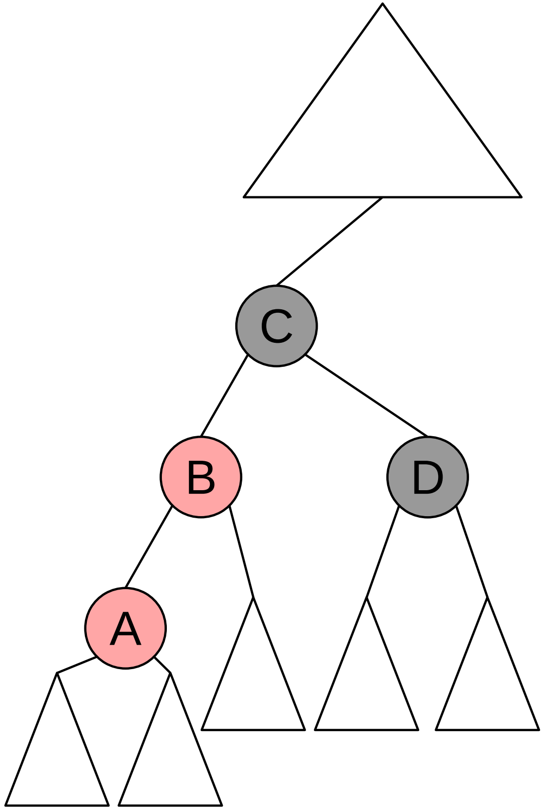
Case 2: Split Black Node



Also works if A is right-child of B (or B is right-child of C)

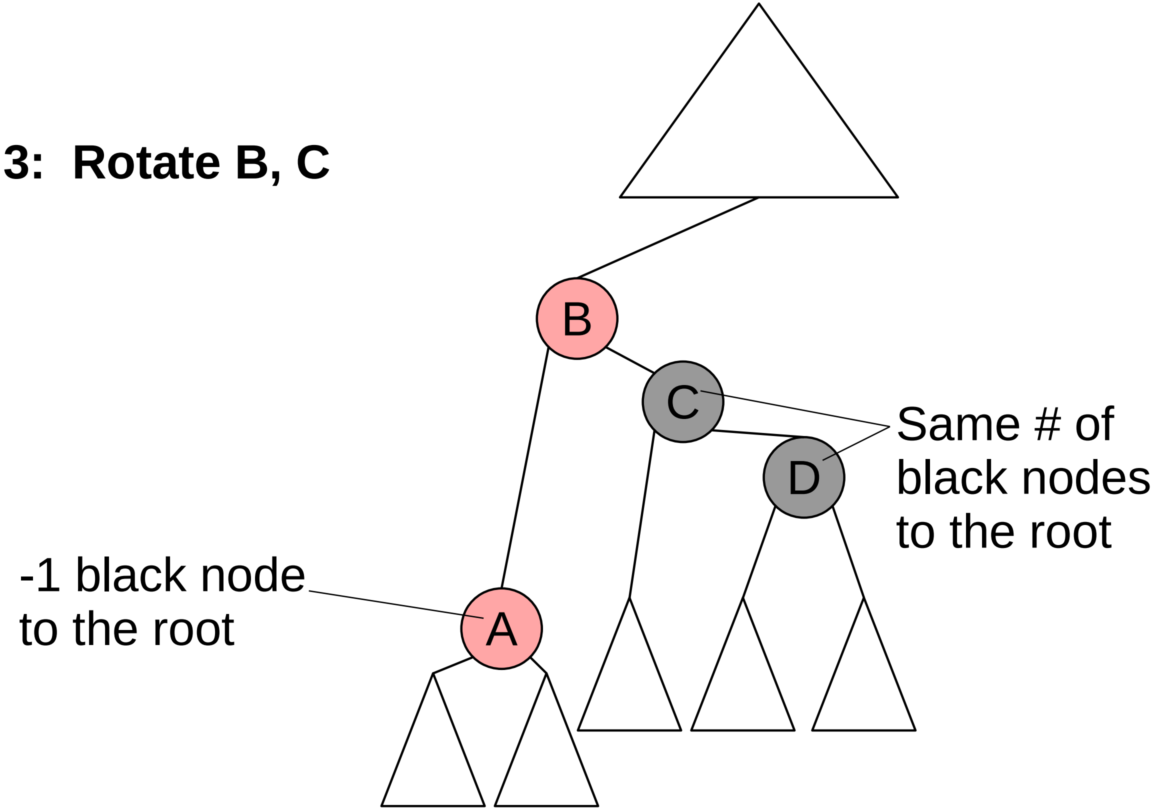
Red-Black Trees

Case 3: Rotate B, C



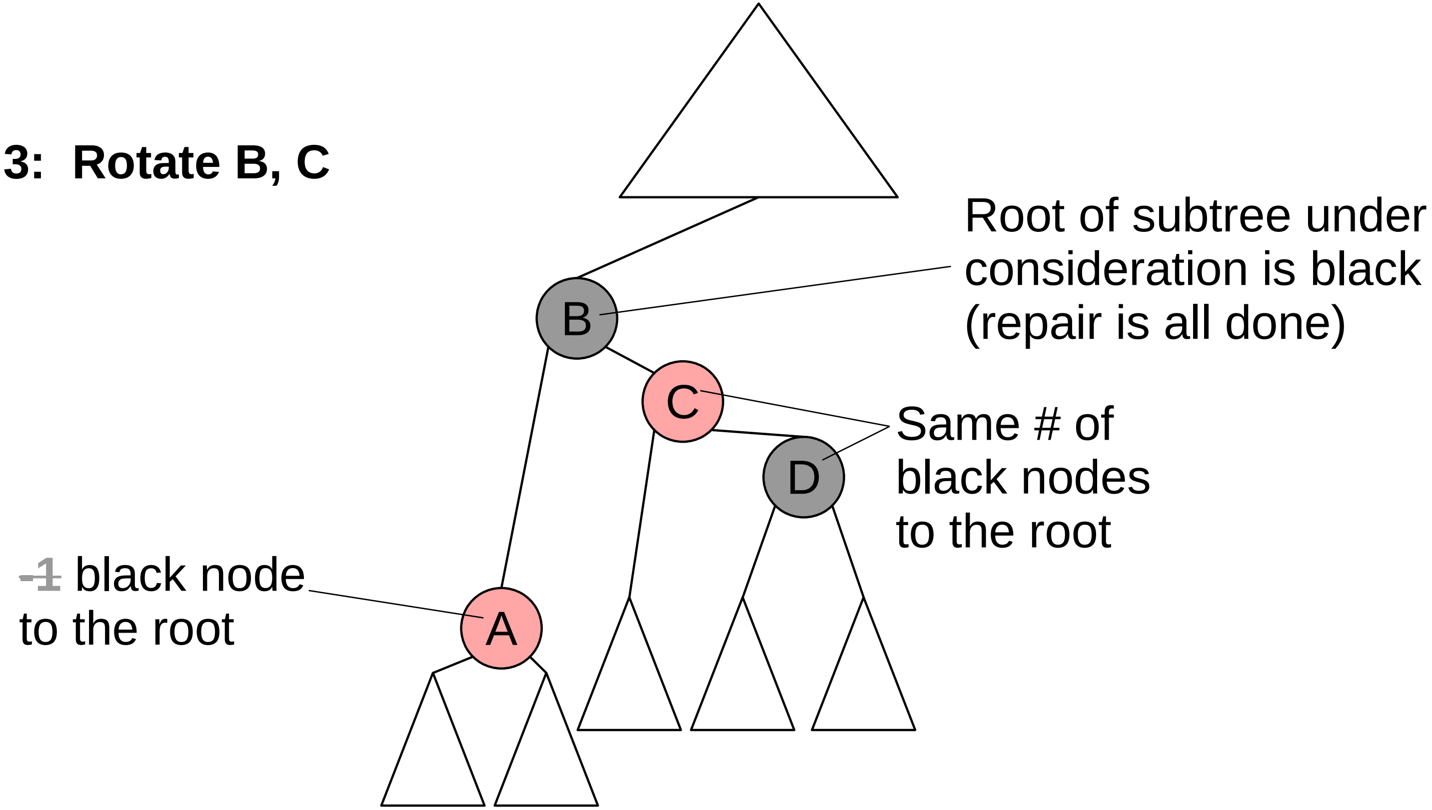
Red-Black Trees

Case 3: Rotate B, C



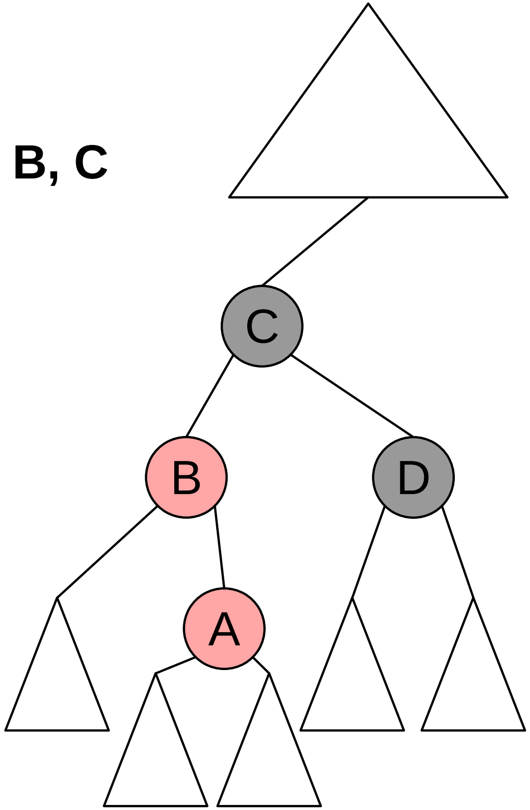
Red-Black Trees

Case 3: Rotate B, C



Red-Black Trees

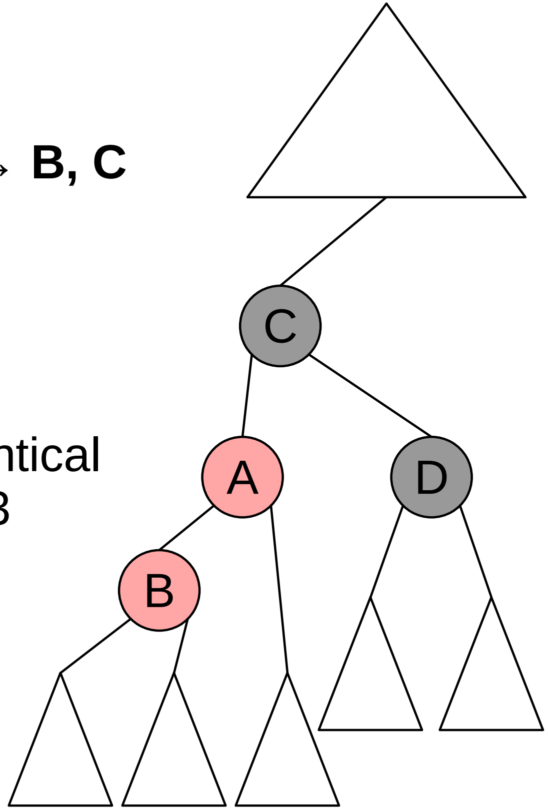
Case 4: Rotate A, B → B, C



Red-Black Trees

Case 4: Rotate A, B → B, C

Now identical
to case 3



Red/Black Trees

- **Case 1** (Parent of Red is Black) **O(1)**
  - Done!
- **Case 1.a** (Root is Red) **O(1)**
  - Recolparent Black **O(1) + O(log(n) * O(1))**
- **Case 2** (Parent is Red; Aunt is Red) ~~**O(1) + fix grandparent**~~
  - Recolor Grandparent Red, Recolor parent and aunt Black
  - Grandparent is now red; Repeat check there
- **Case 3** (Left child of Red Parent; Aunt is Black) **O(1)**
  - Rotate Grantparent Right; Swap rotated node colors
- **Case 4** (Right child of Red Parent; Aunt is Black) **O(1)**
  - Rotate Parent Left; Continue with Case 3

Insertion

- Find the insertion point (as in a BST) **O(d) = O(log(n))**
- Insert the node as red **O(1)**
  - Preserves the black depth
- Fix colors (if needed) **O(log(n))**
  - Preserves the black depth (or adds 1 at root)

Hash Tables

Finding Items: Sequences

- Is it element 1?
  - If so, return, else...
- Is it element 2?
  - If so, return, else...
- Is it element 3?
  - If so, return, else...
- etc...

Finding Items: Sorted Sequences

- How does it compare to element ½ n?
  - If equal, return
  - If lesser, how does it compare to element ¼ n?
    - If equal return
    - If lesser, etc…
    - If greater, etc…
  - If greater, how does it compare to element ¾ n?
    - etc…

Finding Items: Trees

- How does it compare to root?
  - If equal, return
  - If lesser, how does it compare to left child?
    - If equal return
    - If lesser, etc...
    - If greater, etc...
  - If greater, how does it compare to right child?
    - etc...

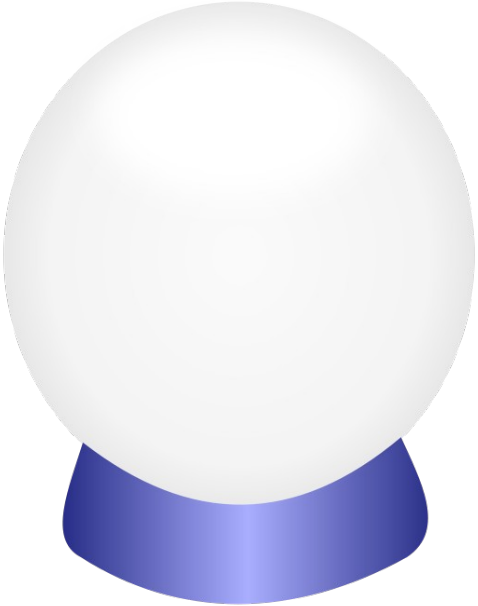
Finding Items

The most expensive part of finding records is **finding** them.

(i.e., where is the record located?)

So... skip the search

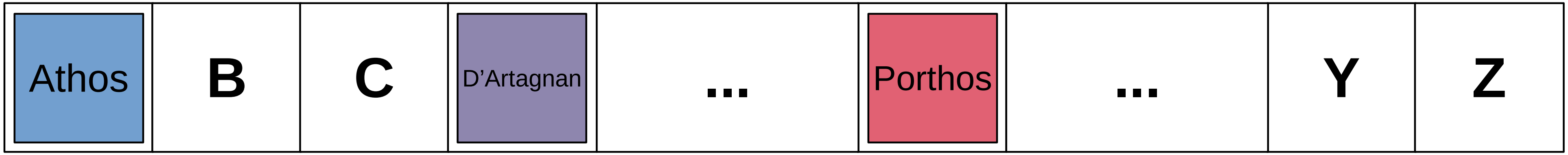
Finding the item



Alternative Idea: Assign Bins

- Create an array of size N
- Pick an O(1) function to assign each record a number in [0,N)
  - First letter of name → [0, 26)

Alternative Idea: Assign Bins



Alternative Idea: Assign Bins

- Pros
  - O(1) Insert
  - O(1) Find
  - O(1) Remove
- Cons
  - Wasted Space (Only 3/26 slots used)
  - Duplication (What about Aramis?)

Other Functions

- Identity Function: `(x: Int) => x`
  - **Problem**: Can return values over N
  - **Solution**: Cap return value by Modulus with N
    - `(x: Int) => x % N`

Other Functions

0	**1**	**2**	**3**	**4**	**5**	**6**

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20

Other Functions

- Identity Function: `(x: Int) => x %N`
- Linear Function: `(x: Int) => (x*a+b) %N` (for some a,b)
- .. or Quadratic: `(x: Int) => ((x*a+b)*x+c) %N` (for a,b,c)

Other Functions

- **Ideal**: Function assigns every record to a unique position
  - If n = N records, every array position is used
  - No conflicted assignment
- Examples
  - Unique Record IDs from [0, N) (like UBIT #s)
    - ... no deletions
  - Cumulative Distribution Functions (CDFs)
    - ... hard to encode

Almost Ideal...

- A function a that evenly distributes records
  - O(1) means we can't compare against other records.
  - **Not random**: Same input = same output
  - **Pseudorandom**: Every position has the same probability
    - (for a given record)
- For n records, the chance of first conflict is n/N
  - Expect $\sqrt{N}$ insertions before the first conflict